OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4722/01**

MATHEMATICS

Core Mathematics 2

TUESDAY 16 JANUARY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

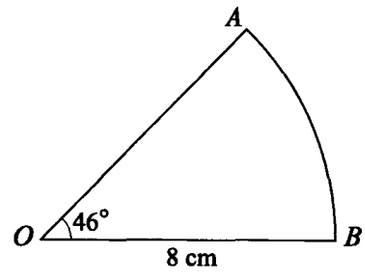
- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 In an arithmetic progression the first term is 15 and the twentieth term is 72. Find the sum of the first 100 terms. [4]

2



The diagram shows a sector OAB of a circle, centre O and radius 8 cm. The angle AOB is 46°.

(i) Express 46° in radians, correct to 3 significant figures. [2]

(ii) Find the length of the arc AB. [1]

(iii) Find the area of the sector OAB. [2]

3 **(i)** Find $\int (4x - 5)\,dx$. [2]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = 4x - 5$. The curve passes through the point (3, 7). Find the equation of the curve. [3]

4 In a triangle ABC, $AB = 5\sqrt{2}$ cm, $BC = 8$ cm and angle $B = 60°$.

(i) Find the exact area of the triangle, giving your answer as simply as possible. [3]

(ii) Find the length of AC, correct to 3 significant figures. [3]

5 **(a)** **(i)** Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$$\int_3^9 \log_{10} x\,dx,$$

giving your answer correct to 3 significant figures. [4]

6 **(i)** Find and simplify the first four terms in the expansion of $(1 + 4x)^7$ in ascending powers of x. [4]

(ii) In the expansion of

$$(3 + ax)(1 + 4x)^7,$$

the coefficient of x^2 is 1001. Find the value of a. [3]

7 **(i)** **(a)** Sketch the graph of $y = 2\cos x$ for values of x such that $0° \leqslant x \leqslant 360°$, indicating the coordinates of any points where the curve meets the axes. [2]

(b) Solve the equation $2\cos x = 0.8$, giving all values of x between 0° and 360°. [3]

(ii) Solve the equation $2\cos x = \sin x$, giving all values of x between −180° and 180°. [3]

8 The polynomial $f(x)$ is defined by $f(x) = x^3 - 9x^2 + 7x + 33$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [1]

(iii) Solve the equation $f(x) = 0$, giving each root in an exact form as simply as possible. [6]

9 On its first trip between Malby and Grenlish, a steam train uses 1.5 tonnes of coal. As the train does more trips, it becomes less efficient so that each subsequent trip uses 2% more coal than the previous trip.

(i) Show that the amount of coal used on the fifth trip is 1.624 tonnes, correct to 4 significant figures. [2]

(ii) There are 39 tonnes of coal available. An engineer wishes to calculate N, the total number of trips possible. Show that N satisfies the inequality

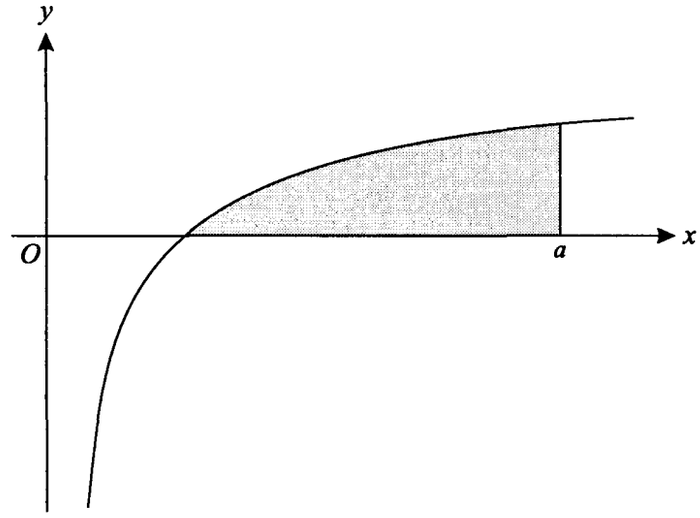
$$1.02^N \leqslant 1.52.$$ [4]

(iii) Hence, by using logarithms, find the greatest number of trips possible. [4]

[Question 10 is printed overleaf.]

[Turn over

10

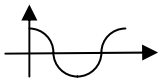


The diagram shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

(i) Verify that the curve intersects the x-axis at (9, 0). [1]

(ii) The shaded region is enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$). Given that the area of the shaded region is 4 square units, find the value of a. [9]

1	$15+19d = 72$ Hence $d = 3$ $S_n = {}^{100}/_{2}\ \{(2\times15)+(99\times3)\}$ $= 16350$	M1 A1 M1 A1	**4**	Attempt to find d, from $a + (n – 1)d$ or $a + nd$ Obtain $d = 3$ Use correct formula for sum of n terms Obtain 16350
			4	
2	**(i)** $46\times\frac{\pi}{180} = 0.802 / 0.803$	M1		Attempt to convert to radians using π and 180 (or 2π & 360)
		A1	**2**	Obtain 0.802 / 0.803, or better
	(ii) 8 x 0.803 = 6.4 cm	B1	**1**	State 6.4, or better
	(iii) ½ x 8^2 x 0.803 = 25.6 / 25.7 cm^2 radians	M1		Attempt area of sector using $½r^2\theta$ or $r^2\theta$, with θ in
		A1	**2**	Obtain 25.6 / 25.7, or better
			5	
3	**(i)** $\int(4x-5)dx = 2x^2-5x+c$	M1		Obtain at least one correct term
		A1	**2**	Obtain at least $2x^2-5x$
	(ii) $y = 2x^2 - 5x + c$ $7 = 2\times3^2-5\times3+c \Rightarrow c = 4$ So equation is $y = 2x^2-5x+4$	B1√ M1 A1	**3**	State or imply y = their integral from **(i)** Use (3,7) to evaluate c Correct final equation
			5	
4	**(i)** area = $\frac{1}{2}\times5\sqrt{2}\times8\times\sin60^o$ $= \frac{1}{2}\times5\sqrt{2}\times8\times\frac{\sqrt{3}}{2}$ $= 10\sqrt{6}$	B1 M1 A1	**3**	State or imply that $\sin60^o = \frac{\sqrt{3}}{2}$ or exact equiv Use $\frac{1}{2}ac\sin B$ Obtain $10\sqrt{6}$ only, from working in surds
	(ii) $AC^2 = \left(5\sqrt{2}\right)^2+8^2-2\times5\sqrt{2}\times8\times\cos60^o$	M1		Attempt to use the correct cosine formula
		A1		Correct unsimplified expression for AC^2
	AC = 7.58 cm	A1	**3**	Obtain AC = 7.58, or better
			6	
5	**(a)** **(i)** $\log_3\frac{4x+7}{x}$	B1	**1**	Correct single logarithm, as final answer, from correct working only
	(ii) $\log_3\frac{4x+7}{x} = 2$			
	$\frac{4x+7}{x} = 9$	B1		State or imply $2 = \log_3 9$
	$4x+7 = 9x$	M1		Attempt to solve equation of form f(x) = 8 or 9
	$x = 1.4$	A1	**3**	Obtain x = 1.4, or exact equiv
	(b) $\int_3^9\log_{10}x\,dx \approx \frac{1}{2}\times3\times\left(\log_{10}3+2\log_{10}6+\log_{10}9\right)$	B1		State, or imply, the 3 correct y-values only
	≈ 4.48	M1		Attempt to use correct trapezium rule
		A1		Obtain correct unsimplified expression
		A1	**4**	Obtain 4.48, or better
			8	

6	**(i)**	$(1+4x)^7 = 1+28x+336x^2+2240x^3$	B1	Obtain 1 + 28x
			M1	Attempt binomial expansion of at least 1 more term, with each term the product of binomial coeff and power of 4x
			A1	Obtain $336x^2$
			A1 **4**	Obtain $2240x^3$
	(ii)	$28a + 1008 = 1001$ Hence $a = -\frac{1}{4}$	M1	Multiply together two relevant pairs of terms
			A1√	Obtain $28a + 1008 = 1001$
			A1 **3**	Obtain $a = -\frac{1}{4}$
			7	
7	**(i)**	**(a)** 	B1	Correct shape of $k\cos x$ graph
			B1 **2**	(90, 0), (270, 0) and (0, 2) stated or implied
		(b) $\cos x = 0.4$ $x = 66.4^o, 294^o$	M1	Divide by 2, and attempt to solve for x
			A1	Correct answer of 66.4^o / 1.16 rads
			A1√ **3**	Second correct answer only, in degrees, following their x
	(ii)	$\tan x = 2$	M1	Use of $\tan x = \frac{\sin x}{\cos x}$ (or square and use $\sin^2 x + \cos^2 x \equiv 1$)
		$x = 63.4^o, -117^o$	A1	Correct answer of 63.4^o / 1.56 rads
			A1√ **3**	Second correct answer only, in degrees, following their x
			8	
8	**(i)**	$-8 - 36 - 14 + 33 = -25$	M1	Substitute $x = -2$, or attempt complete division by $(x + 2)$
			A1 **2**	Obtain –25, as final answer
	(ii)	$27 - 81 + 21 + 33 = 0$ **A.G.**	B1 **1**	Confirm f(3) = 0, or equiv using division
	(iii)	$x = 3$	B1	State $x = 3$ as a root at any point
		$f(x) = (x - 3)(x^2 - 6x - 11)$	M1	Attempt complete division by $(x - 3)$ or equiv
			A1	Obtain $x^2 - 6x + k$
			A1	Obtain completely correct quotient
		$x = \frac{6 \pm \sqrt{36+44}}{2}$	M1	Attempt use of quadratic formula, or equiv, to find roots
		$= 3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$	A1 **6**	Obtain $3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$
			9	
9	**(i)**	$u_5 = 1.5 \times 1.02^4$	M1	Use $1.5r^4$, or find u_2 , u_3 , u_4
		= 1.624 tonnes **A.G.**	A1 **2**	Obtain 1.624 or better
	(ii)	$\frac{1.5(1.02^N - 1)}{1.02 - 1} \leq 39$	M1	Use correct formula for S_N
			A1	Correct unsimplified expressions for S_N
		$(1.02^N - 1) \leq (39 \times 0.02 \div 1.5)$	M1	Link S_N to 39 and attempt to rearrange
		$(1.02^N - 1) \leq 0.52$		
		Hence $1.02^N \leq 1.52$	A1 **4**	Obtain given inequality convincingly, with no sign errors
	(iii)	$\log 1.02^N \leq \log 1.52$	M1	Introduce logarithms on both sides and use $\log a^b = b \log$
		$N \log 1.02 \leq \log 1.52$	A1	Obtain $N \log 1.02 \leq \log 1.52$ (ignore linking sign)
		$N \leq 21.144..$	M1	Attempt to solve for N
		$N = 21$ trips	A1 **4**	Obtain $N = 21$ only
			10	

10	**(i)** $0 = 1 - \frac{3}{\sqrt{9}}$	B1	**1**	Verification of (9, 0), with at least one step shown
	(ii) $\int_9^a 1 - 3x^{-\frac{1}{2}} dx = \left[x - 6\sqrt{x}\right]_9^a$	M1		Attempt integration – increase in power for at least 1 term
		A1		For second term of form $kx^{½}$
		A1		For correct integral
	$= \left(a - 6\sqrt{a}\right) - \left(9 - 6\sqrt{9}\right)$	M1		Attempt F(a) – F(9)
	$= a - 6\sqrt{a} + 9$	A1		Obtain $a - 6\sqrt{a} + 9$
	$a - 6\sqrt{a} + 9 = 4$	M1		Equate expression for area to 4
	$a - 6\sqrt{a} + 5 = 0$	M1		Attempt to solve 'disguised' quadratic
	$\left(\sqrt{a} - 1\right)\left(\sqrt{a} - 5\right) = 0$			
	$\sqrt{a} = 1, \sqrt{a} = 5$	A1		Obtain at least $\sqrt{a} = 5$
	$a = 1$, $a = 25$			
	but $a > 9$, so $a = 25$	A1	**9**	Obtain $a = 25$ only
			10	